The Residual Strength of Light Alloy Sheets Containing Fatigue Cracks

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SUMMARY

A survey is made of the state of the art of the residual strength problem. Criteria are presented for crack growth and fracture, which are confirmed by test results. Test data are given of systematic investigations into the effects of sheet width, sheet thickness, load release and stop holes on the residual strength. Some residual strength data of large wing panels are presented and discussed.

1. INTRODUCTION

The fail-safe design philosophy admits the possibility of the initiation of fatigue cracks in fail-safe structural components. With this philosophy, apart from information on the fatigue crack propagation properties, knowledge of the effect of cracks on the residual static strength is required. Fail-safe design is generally limited to sheet structures, a reason why the scope of the present paper is also limited to sheet and sheet structures.

At the National Aerospace Laboratory NLR at Amsterdam an extensive test programme has been carried out to investigate the various parameters affecting the residual strength of cracked sheet. A summary of the results obtained from these tests will be presented in this paper. The presentation of test data will be preceded by some theoretical considerations which will facilitate the interpretation of the test results. Finally, a conclusion on the usefulness of data obtained from unstiffened sheet for application to stiffened sheet structures will be drawn. Some residual strength data of large wing structures will be presented.

The scope of the paper is limited to aluminium alloys, which are still the most widely used materials for aircraft structures. The general trends emerging from this paper, however, will apply to other materials as well.

SYMBOLS

E	modulus of elasticity
$2l$	crack length
$2l_0$	initial crack length
$2l_c$	critical crack length at fracture
P	potential energy
r_p	width of plastic zone at crack tip
t	sheet thickness
U	elastic energy
W	plastic energy
$2w$	specimen width
α, β	constants
σ	tensile stress
σ_c	critical (fracture) stress (residual strength)
σ_i	stress to initiate crack growth
σ_u	ultimate tensile strength
σ_y	yield stress
$\sigma_1, \sigma_2, \sigma_3$	principal stresses
τ	shear stress

All stresses are based on gross area.

2. THE BEHAVIOUR OF A CRACKED SHEET UNDER STATIC LOAD

Consider a sheet containing a central transverse crack loaded in tension (Fig. 1) by a gradually increasing load. At a particular gross stress the crack will start to extend slowly. This slow crack growth is stable: it stops immediately when the load is kept constant. Crack growth can be maintained only by gradually increasing the load; as the crack grows in length a higher stress is required to keep it growing. At a certain critical stress a critical crack length is reached, where crack growth becomes unstable and a sudden total fracture of the sheet is the result. When the initial crack is longer, crack growth can start at a lower stress and also the fracture stress (residual strength) is lower, although there is more slow crack growth (Fig. 1).

The slow crack growth presents a technical problem since an important question is, what is essential for predicting the residual strength, either the initial crack length or the critical crack length or perhaps both? Slow crack growth is also a difficulty in fracture mechanics and until now there is no satisfactory theory accounting also for slow crack growth. In fracture mechanics two approaches have been tried to cope with the residual strength problem. The first is the classical energy balance criterion, presented as early

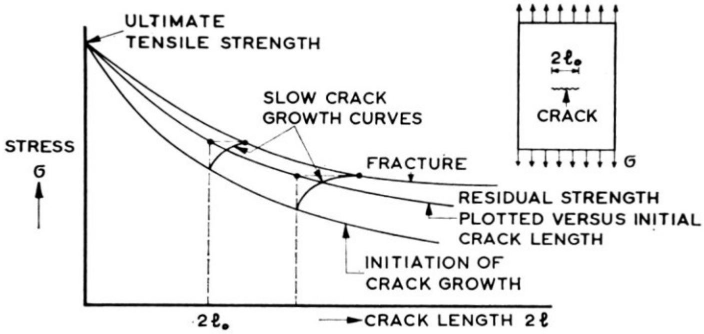


FIG. 1 — Residual strength characteristic

as 1921 by Griffith[1] and later modified by Irwin.[2] The other approach considers the peak stress at the tips of the crack as responsible for fracture. Both treatments lead to the well-known K_c-concept[3].

It will be shown that a phenomenological approach, concerning slow crack growth, leads to a combined stress and energy criterion for crack growth and a modified energy criterion for fracture, which are in reasonable agreement with test results. It cannot be stated that this treatment is fully satisfactory, but it is felt that it can give a basis for further development, since it can at least qualitatively explain and correlate certain phenomena observed in tests.

3. The Stress Criterion for the Onset of Crack Growth

It is assumed that crack growth cannot occur unless the peak stress at the tip of the crack exceeds a certain critical value. Calculations of the elastic stress field at the tip of a transverse crack in an infinite sheet can be found in text books of applied mechanics. They lead to the following expressions:

$$\left.\begin{aligned} \sigma_x &= \sigma\sqrt{(l)}f(r)f_1(\theta) \\ \sigma_y &= -\sigma+\sigma\sqrt{(l)}f(r)f_2(\theta) \\ \tau_{xy} &= \sigma\sqrt{(l)}f(r)f_3(\theta) \end{aligned}\right\} \qquad (1)$$

where r and θ are the polar co-ordinates (originating at the tip of the crack) of a point where σ_x and σ_y are the stresses in the longitudinal and the transverse direction, respectively. The stress σ is the uniform stress in the longitudinal direction far from the crack and l is the semi-crack length.

Equations (1) show that (apart from the unimportant term $-\sigma$ in the second equation) the stress field at the tip of the crack can be described by the parameter $\sigma\sqrt{l}$. The stress criterion for crack growth can thus be based on this parameter. For the initiation of the growth of a crack of initial length $2l_0$ it is assumed that the value of $\sigma\sqrt{l}$ exceeds a critical value. Hence,

$$\sigma_i \sqrt{l_0} = C = \text{constant} \tag{2}$$

is the criterion for the onset of crack growth, where σ_i is the stress required to initiate crack growth.

During slow crack growth the crack length increases and the stress required to maintain crack growth increases. This means that the resistance to crack growth increases. It is difficult to see, however, what the stress criterion for fracture instability would be. Therefore, an attempt will be made in section 4 to find the fracture condition from an energy balance criterion.

4. The Energy Criterion for Crack Growth and Fracture

Consider a large sheet of unit thickness with a central transverse crack of semi-length l. When the crack grows by an amount $\mathrm{d}l$ an amount of energy $(\mathrm{d}W/\mathrm{d}l)\mathrm{d}l$ is consumed, which will be written as $W'(l)\mathrm{d}l$ because the integral W itself is a function of l only. The major part of $W'(l)$ consists of energy for plastic deformation required to produce a new plastically deformed zone at the tip of the advancing crack. The energy $W'(l)$ has to be generated by a release of potential energy $(\mathrm{d}P/\mathrm{d}l)\mathrm{d}l$. The crack growth is stable if the released energy is just balanced by the energy consumption, *i.e.* the condition for slow crack growth will read:

$$\frac{\mathrm{d}P}{\mathrm{d}l} + W'(l) = 0 \tag{3}$$

Unstable crack growth or fracture instability can occur when

$$-\frac{\mathrm{d}P}{\mathrm{d}l} > W'(l) \tag{4}$$

It can be shown (*see* appendix) that the energy release consists of elastic energy and that

$$-\frac{\mathrm{d}P}{\mathrm{d}l} = -\frac{\partial U}{\partial l} = \frac{2\pi\sigma^2 l}{E} \tag{5}$$

where U is the elastic energy of the sheet, σ the gross stress and E the modulus of elasticity. According to eqns. (3) and (5), during slow crack growth

$$W'(l) = -\frac{\partial U}{\partial l} = \frac{2\pi\sigma^2 l}{E} \tag{6}$$

During slow crack growth both σ and l increase, which implies that the rate of energy consumption $W'(l)$ (eqn. 6) increases during crack growth. From simultaneous values of σ and l obtained from cinematographic records of slow crack growth, $W'(l)$ could be calculated with the aid of eqn. (6). The result is a curve as shown diagrammatically in Fig. 2. The increase of $W'(l)$

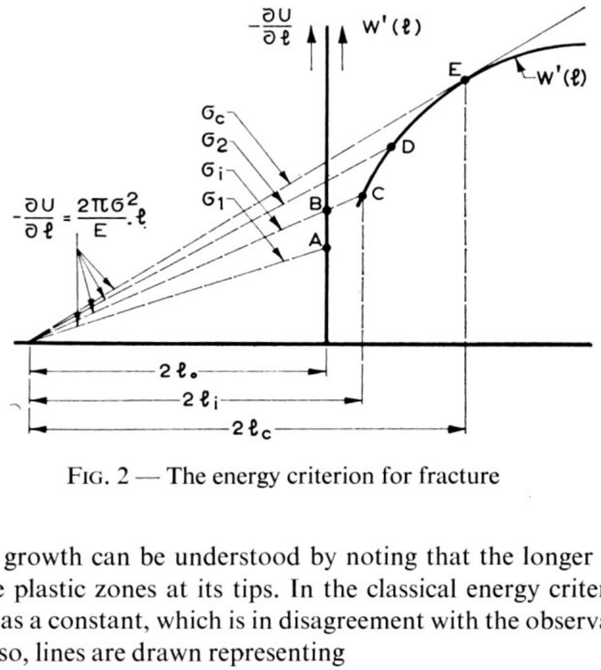


FIG. 2 — The energy criterion for fracture

during crack growth can be understood by noting that the longer the crack, the larger the plastic zones at its tips. In the classical energy criterion $W'(l)$ is considered as a constant, which is in disagreement with the observations.

In Fig. 2 also, lines are drawn representing

$$-\frac{\partial U}{\partial l} = \frac{2\pi\sigma^2 l}{E}$$

(eqn. (5), which are straight lines as a function of l, the slope of the lines being proportional to σ^2. Let $2l_0$ be the length of the initial crack. At a stress σ_1 the energy release rate $-\partial U/\partial l$ would be represented by point A in Fig. 2, but this value of $-\partial U/\partial l$ is hypothetical since there is still no crack growth. Suppose slow crack growth starts when the stress is raised to σ_i (stress criterion of the previous section). Initially the value of $-\partial U/\partial l$ is represented by point B (Fig. 2), but the crack can now extend to the length $2l_i$ where $W'(l) = -(\partial U/\partial l)$, *i.e.* released and consumed energy are balanced. The extension of the crack from $2l_0$ to $2l_i$ is the well-known 'pop-in' phenomenon. During a further gradual increase of the stress to σ_2 the crack grows slowly to the length $2l_2$. When the stress is raised to σ_c, the crack has reached a length $2l_c$ (Fig. 2). The line for $\partial U/\partial l$ belonging to σ_c is tangent to the curve for $W'(l)$. This means that the crack can now grow under constant stress. Moreover, there will be an increasing over-supply of energy, indicating that fracture instability occurs. Then apparently the fracture criterion reads (point of tangency):

$$\left.\begin{aligned}\frac{\partial U}{\partial l}+W'(l)=0\\ \frac{\partial^2 U}{\partial l^2}+W''(l)=0\end{aligned}\right\} \tag{7}$$

The evaluation of the fracture criterion of eqns. (7) is possible in a semi-empirical way. From tests on wide specimens with short cracks it may be concluded (Figs. 10, 11) that for an infinite sheet the critical crack length is proportional to the initial crack length, *i.e.*

$$l_c=\alpha l_0 \tag{8}$$

It is further assumed that the function $W'(l)$ is the same for any value of the initial crack length or, in other words, that the curve for $W'(l)$ in Fig. 2 is invariant for a certain material, which was first proposed by Krafft *et al*[5].

Then eqn. (8) requires a distinct behaviour of $W'(l)$, which according to the appendix can be given by

$$W'(l)=\beta(l-l_0)^{(\alpha-1)/\alpha} \tag{9}$$

where β is a constant.

Equations (5) and (9) make it possible to evaluate the fracture criterion of eqns. (7). The result is

$$\sigma_c l_c{}^{1/2\alpha}=\text{constant} \tag{10}$$

and also

$$\sigma_c l_0{}^{1/2\alpha}=\text{constant}. \tag{11}$$

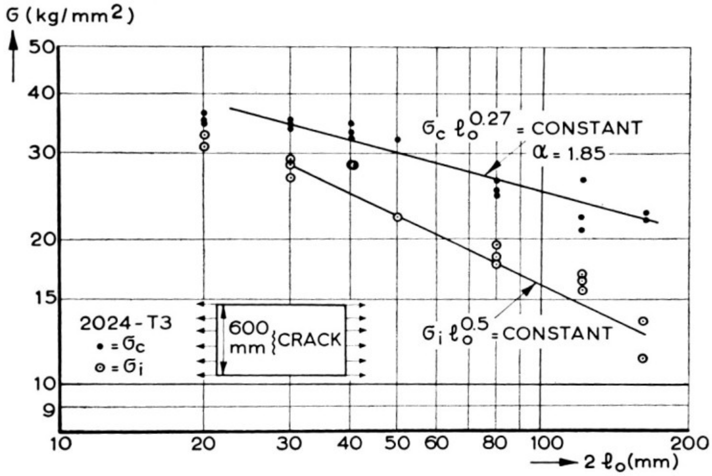
Eqn. (11) is interesting, because it gives a direct relation between the initial crack length and the residual strength.

It can be seen that the minimum value for α is unity. Then there is no slow crack growth according to eqn. (8) and immediate fracture occurs. (For $\alpha=1$ eqns. (2) and (11) are thus equivalent: $\sigma_i=\sigma_c$.)

$\alpha=1$ for an ideally brittle material.

5. Test Data Confirming the Criteria for Crack Growth and Fracture

The criteria developed in the previous sections are valid only for an infinite sheet. Consequently, these criteria should be checked with test data obtained from large specimens with short cracks. Test results of 600 mm wide specimens containing cracks smaller than 20% of the specimen width are shown in Fig. 3. In Fig. 3 the relations of eqns. (2) and (11) are straight lines. Both the



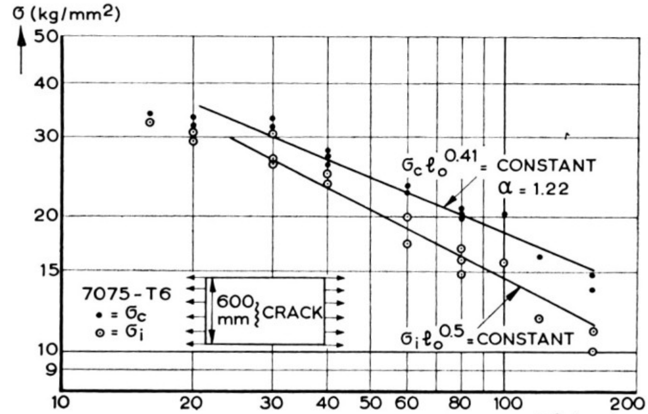


Fig. 3 — Check of crack growth and fracture criteria (Test results of Ref. 6)
(*a*) 2024 – T3 material (*b*) 7075 – T6 material

criterion for the onset of crack growth and the criterion for fracture are in reasonable agreement with the test data in Fig. 3. (Compare also the values for α with those found in Figs. 10 and 11.) There are deviations from the straight lines at large cracks, which may be due to the effect of the finite specimen size. There are also deviations from the straight lines at very small cracks. The criteria of eqns. (2) and (11) predict infinitely high stresses at zero crack length. For $l=0$, however, the residual strength must be equal to the ultimate tensile strength. This discrepancy can be explained by noting that for short cracks the strength is high enough to give general yielding in the cross-section containing the crack; then, of course, the expressions for $W'(l)$ and $\partial U/\partial l$ are meaningless.

Results of tests on specimens with blunted crack tips are very informative for the crack growth criteria. Results of specimens with fatigue cracks, saw-cut simulated cracks and cracks with stop holes are presented in Fig. 4.†

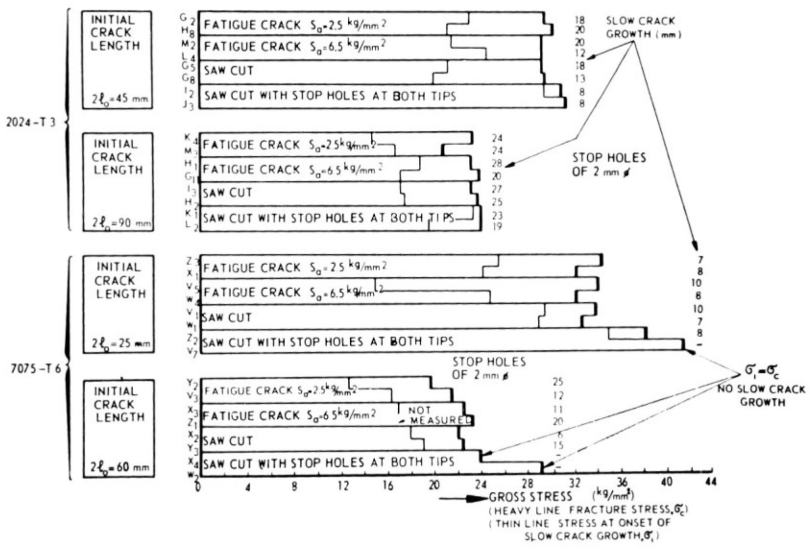


FIG. 4 — Stresses for crack growth and fracture[7]

In 2024–T3 material all types of cracks resulted in the same residual strength, although the stress for the initiation of crack growth was generally higher for a blunter crack. For 7075–T6 material stop holes increased the residual strength; there was no, or almost no slow crack growth when stop holes were present.

Consider first a crack with small stop holes at its tips. Due to the lower

† Each horizontal bar represents two duplicate tests.

stress concentration at this blunt crack, a higher nominal stress is required to initiate crack growth than for a fatigue crack. Once crack growth has started, the difference with any other crack disappears and the behaviour will be the same as in the fatigue crack. This is illustrated in Fig. 5. For the speci-

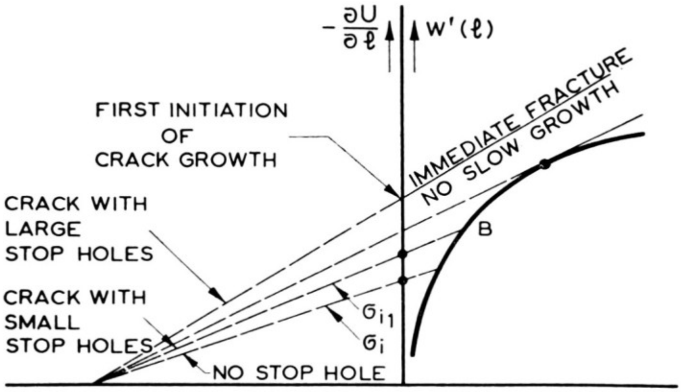


FIG. 5 — Behaviour of a crack with stop holes

men with stop holes the onset of crack propagation is postponed to a higher stress. At this higher stress the crack extends to point B (Fig. 5), where $W'(l)$ and $-\partial U/\partial l$ are balanced. From then on the behaviour is as if there had been no stop holes. Stop holes can be made so large that the stress for the initiation of crack growth exceeds the fracture stress σ_c for a fatigue crack of the same initial length. When crack growth starts at this high stress no energy balance can be obtained (Fig. 5) and immediate fracture will occur; there is no slow crack growth. The latter case occurred in the tests on 7075–T6 in Fig. 4. It can thus be concluded that stop holes must have a certain minimum size to have any effect on the residual strength at all. More ductile materials will require larger stop holes, since these materials are able to reduce the stress concentration by plastic flow more effectively than a brittle material (compare 2024–T3 and 7075–T6 in Fig. 4).

It will be clear now why fine saw cuts can so well simulate fatigue cracks for the purpose of residual strength tests. The stress to initiate slow crack growth will be slightly higher for a saw cut, but reliable values for residual strength and critical crack length are obtained. Very brittle materials will be more sensitive to a variation in acuteness of the crack tip and for certain materials a saw cut cannot be used for a residual strength test.

Another indication for the influence of the stress at the crack tip is presented by the results of interrupted tests. Consider a specimen with an initial crack $2l_0$. This specimen is loaded to a stress σ_r (Fig. 6) at which the crack has reached a length $2l_1$. Now, the load is released to zero. At subsequent reloading the extreme cases that might occur are:

(*a*) The specimen behaves like a new specimen with an initial crack length $2l_1$. Crack growth starts at a stress σ_{i1} (Fig. 6) and fracture occurs at a stress σ_{c1}.

(*b*) The specimen ignores the unloading and crack growth is continued when the stress σ_r is reached at which the test was interrupted.

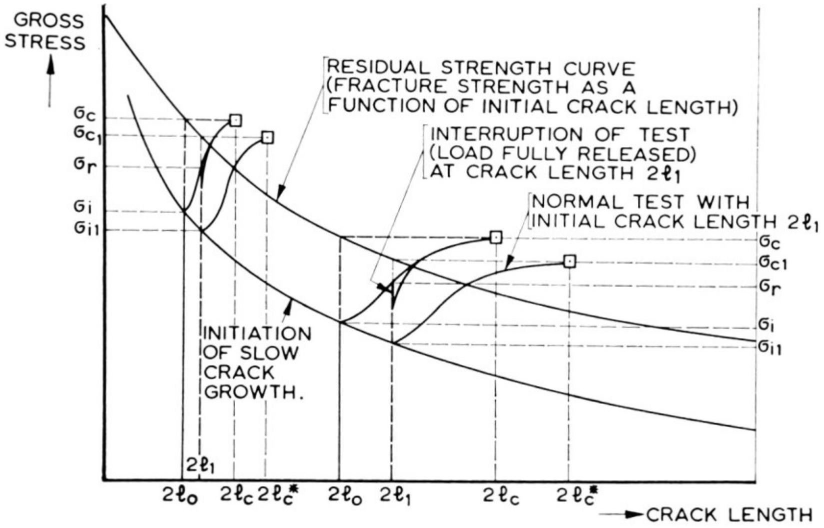


FIG. 6 — Possible crack propagation curves at reloading after interruption of residual strength tests

From interrupted tests carried out on 600 mm wide specimens[8] it may be concluded that the actual behaviour approaches case (*b*), as is shown in Fig. 7. The specimen practically ignores the unloading, although the re-initiation of crack growth occurs at a stress somewhat below that at which the test was interrupted. The stress at the crack tip must be responsible for this phenomenon, as can be appreciated from Fig. 8. At the moment of unloading, a zone of plastically deformed material is present at the crack tip. If unloading were fully elastic (Fig. 8(*b*)), reloading to the stress σ_r would be required to restore the stress distribution of Fig. 8(*a*) and, hence, to restore the stress at the crack tip required to maintain crack growth. However, unloading is not fully elastic; reversed plastic straining occurs as indicated in Fig. 8(*c*). At reloading the stress at the crack tip will reach the value of Fig. 8(*a*) at a nominal stress somewhat lower than σ_r. Then it may be expected that the re-initiation of crack growth also takes place at a stress below σ_r. When the crack has grown through the small region of reversed plastic flow the unloading is no longer effective and the behaviour will be the same as in a

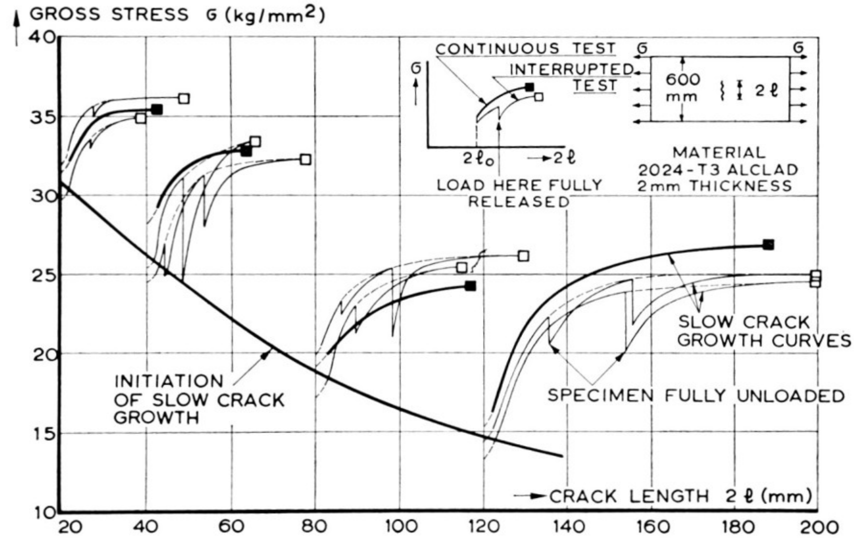


FIG. 7 — Influence of unloading to zero stress on slow crack growth and strength[8]

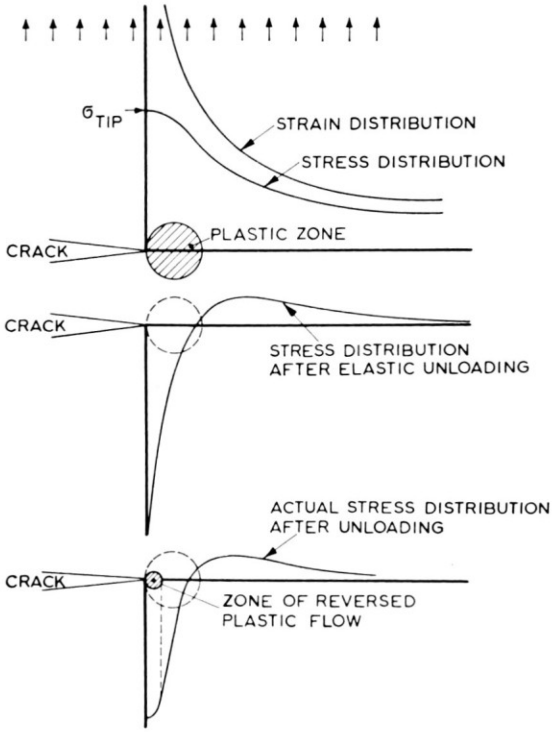


FIG. 8 — Stress distribution at crack tip after unloading

continuous test. It seems reasonable to conclude that if the interrupted tests had been carried out as continuous tests, crack propagation curves would have been obtained similar to the dotted curves in Fig. 7.

This result is also of practical importance, because a high load on an aircraft may reach a level at which a crack in the structure shows slow growth. Although the crack is longer now, the residual strength is not impaired. (Since the high load will introduce favourable residual stresses at the crack tip (Fig. 8(*b*)) fatigue crack propagation during subsequent fatigue loading will be slowed down, ref. 9).

6. Usefulness of Test Data and Theory

It seems reasonable to conclude from the foregoing that crack growth can occur when the stress at the crack tip exceeds a certain critical value. Crack growth is stable as long as the energy release rate $-\partial U/\partial l$ is balanced by the

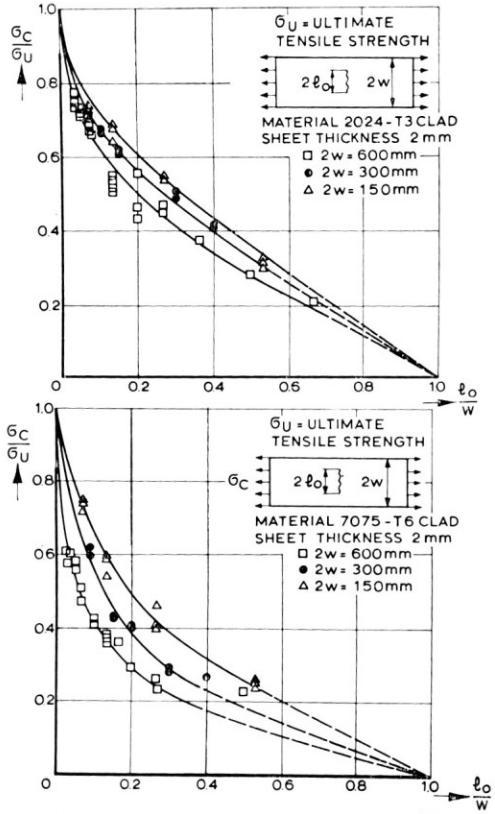


FIG. 9 — Influence of specimen size[6]

energy consumption rate $W'(l)$. Fracture instability occurs when the released energy exceeds the consumed energy and the fracture condition can then be given by $\partial^2 U/\partial l^2 + W''(l) = 0$. Although the evaluation of these criteria as presented in the previous sections certainly does not constitute a rigorous proof, it is felt that the analysis gives a good starting point for further developments.

The evaluation of the fracture criterion was made for an infinite sheet. It is possible by applying a correction factor to $-\partial U/\partial l$ to extend the evaluation to a specimen of finite size[6], but this treatment does not account properly for the effect of finite width. The effect of finite width is appreciable, as can be seen from the test results presented in Fig. 9, plotted on the basis of the relative crack length l/w ($2w$=specimen width). This figure shows that the residual strength is not fully determined by the relative crack length l/w, but that also the absolute crack length is of importance. (For a particular value of l/w the largest specimen, *i.e.* the specimen with the longest crack, has the lowest strength.)

The test results of small specimens and of specimens with large cracks cannot satisfactorily be explained by the fracture criterion. It must be noted,

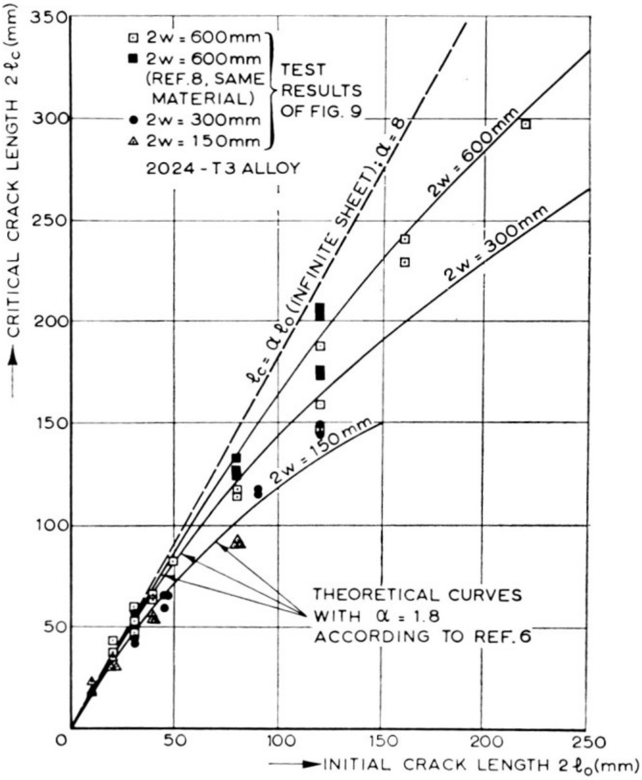


FIG. 10 — Relation between initial crack length and critical crack length

however, that for aircraft structures generally wide sheets are used, such as wing skins or tailplane skins. Furthermore, large cracks cannot be tolerated in practice and, therefore, in general only relatively small cracks in wide sheets will be of interest. From Fig. 3 it may be concluded that then the panel can be treated as an infinite sheet.

On the other hand, it can be stated that test data obtained from small specimens are of limited value for design applications. Such data cannot give reliable figures for the residual strength, but they can serve as a guide for the choice of the structural material. When complete residual strength curves are not available it is probably better to base the decisions upon the amount of slow crack growth. According to eqns. (8) and (11) and Figs. 10 and 11 a large amount of slow crack growth (α large) is an indication for good residual strength characteristics.

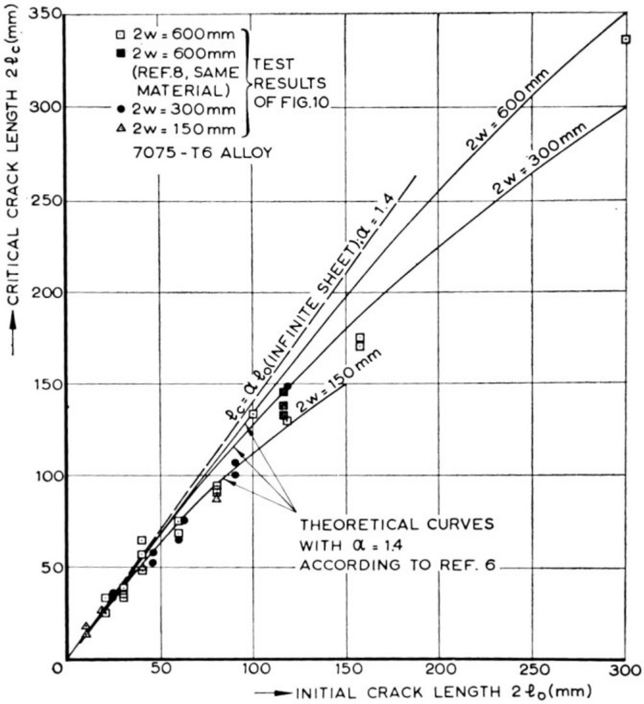


FIG. 11 — Relation between initial crack length and critical crack length

7. The Effect of Sheet Thickness

The residual strength of a sheet specimen depends on the thickness of the sheet. The effect of sheet thickness is demonstrated by the test results in

Figs. 12 and 13. The residual strength has a maximum at a particular thickness. Such a maximum was found by various investigators. The maximum here is at a thickness of 2 mm. For β-titanium it is at about 0·3 mm thickness[11]. This optimum thickness depends on the material properties, as will become clear in the following.

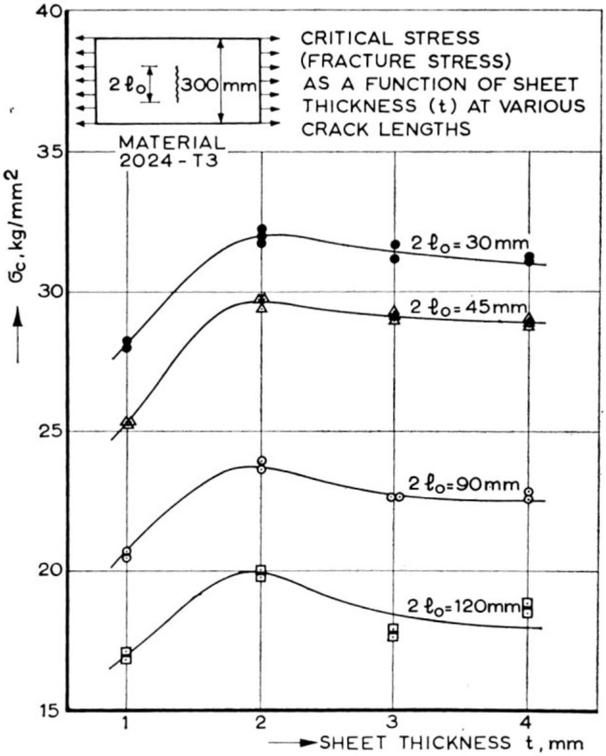


FIG. 12 — Influence of sheet thickness on residual strength

Specimens in the ascending part of the curve (Fig. 13) have a fracture surface that is slant under an angle of 45° with the sheet surface. This is the well-known shear mode fracture. Beyond the maximum in the residual strength curve a portion of the fracture surface at mid-thickness is perpendicular to the sheet surface (Fig. 13). This portion, called the flat tensile part, is larger when the specimen is thicker and in very thick specimens the fracture surface is almost completely of the flat tensile mode with only small shear lips (Fig. 13).

The fracture is related with the state of stress at the crack tip. In thin sheets plastic deformation at the crack tip can take place freely and the result is a

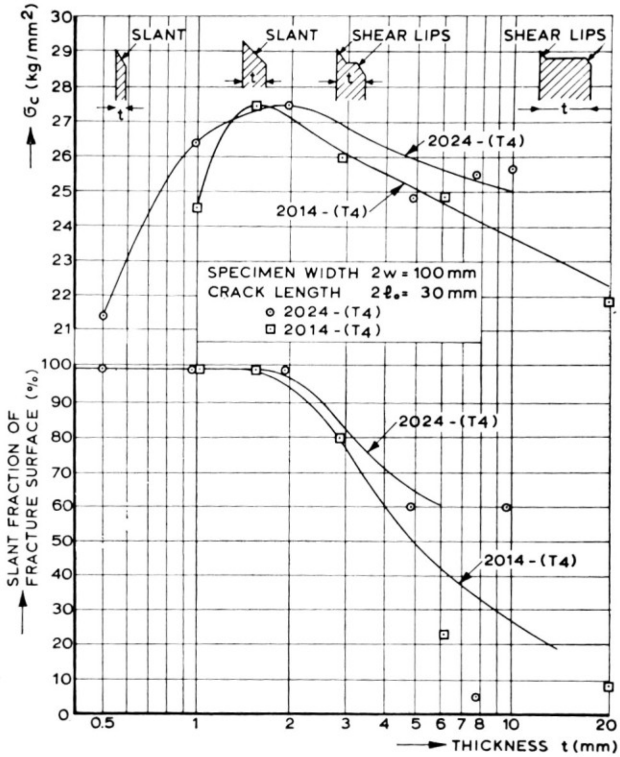


FIG. 13 — Influence of thickness on residual strength and fracture mode

state of plane stress giving rise to a slant fracture. When the sheet is thicker, yielding at the crack tip is constrained by the surrounding elastic material and a triaxial state of stress exists at the crack tip, which is ultimately a state of plane strain, giving rise to a flat tensile fracture. Plane strain will occur at mid-thickness and the thicker the sheet the larger the region with plane strain, which correlates with a larger flat tensile fracture portion. The third principal stress cannot exist at the sheet surface, which explains why shear lips are always present.

An explanation of the relation of the fracture appearance and the residual strength with the state of stress has been given in ref. 10. A summary will be given here. From electron-microscopical observations of fracture surfaces it has appeared that static ductile fracture is a result of initiation, growth and coalescence of micro-voids. The halves of these voids can be seen in electron-micrographs (Fig. 14). There is reason to believe(10) that void initiation and growth is a result of slip (plastic shear deformation). For this reason voids will occur most abundantly on the planes of maximum shear stress. The

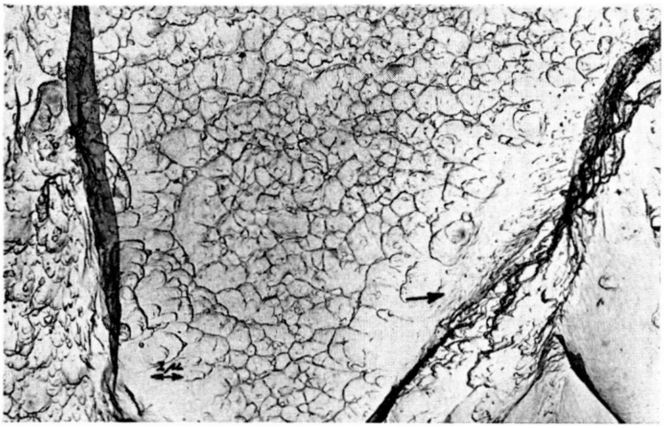


FIG. 14 — Electron micrograph of ductile fracture surface. The picture shows 'dimples' which are the halves of micro-voids. Arrow points to smoothed slip steps

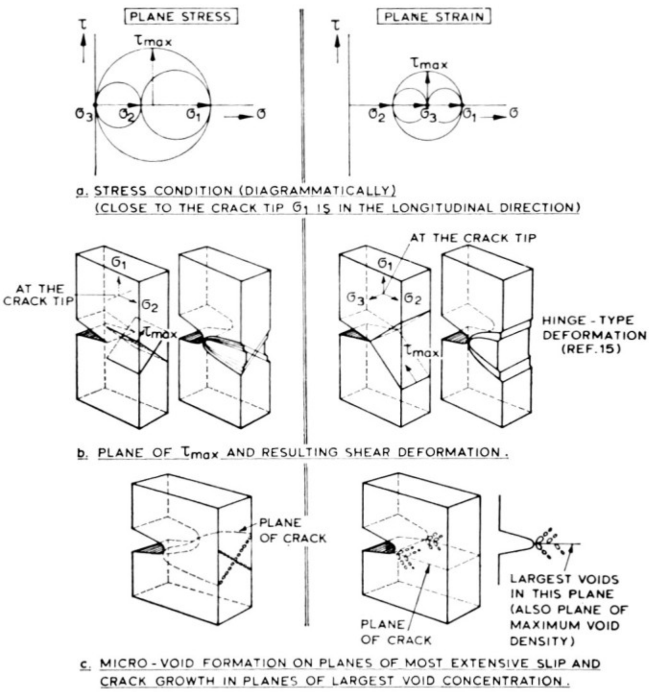


FIG. 15 — Slant and square fractures as a result of the state of stress

planes of maximum shear stress are different in the case of plane stress and in plane strain. According to ref. 10 the state of stress in the two cases will be very close to what is diagrammatically shown in Fig. 15(*a*). From Fig. 15(*a*) it follows that in plane stress the maximum shear stress at the crack tip is on planes making an angle of 45° with the sheet surface. For plane strain the maximum shear stress is on planes perpendicular to the sheet surface, but making an angle of about 45° with the loading direction (Fig. 15(*b*)). This results in the planes of maximum void concentration shown in Fig. 15(*b*). The crack will follow a path through the regions with the largest void concentration, which leads to a flat tensile fracture in the case of plane strain and a slant fracture in plane stress.

A tentative explanation can be offered for the influence of sheet thickness on the residual strength. Figure 16 shows three mechanisms of void coalescence proposed by Beachem[12] on the basis of electron-microscopical observations. It may be concluded from Fig. 16 that the mechanisms of void coalescence are dominated by tensile stresses. Of course, void coalescence in itself will probably still be a slip mechanism, but in planes of large void concentrations the stress distribution is very complicated and it must be concluded that the shear stresses responsible for void coalescence are governed by the tensile stress acting on the plane of maximum void concentration. Then it must be concluded that void coalescence and, hence, crack growth

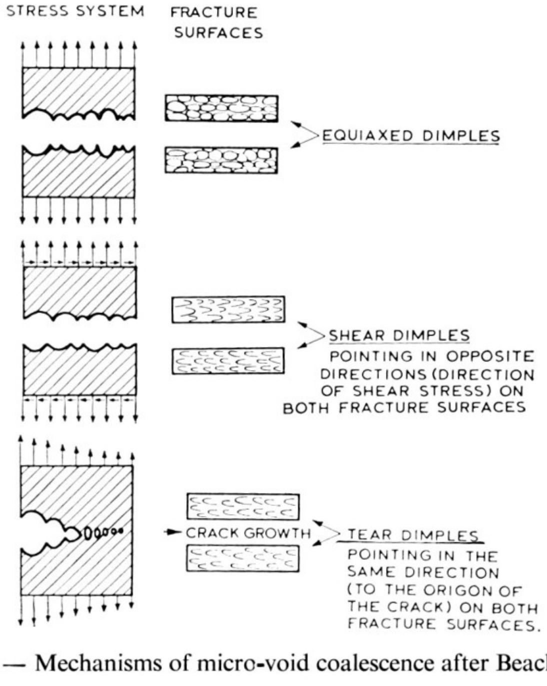


FIG. 16 — Mechanisms of micro-void coalescence after Beachem[12]

are governed by this tensile stress, a point of view which correlates with the stress criterion for crack growth in sections 3 and 5.

Consider now Fig. 17. In the lower line of this figure five specimens are compared of thicknesses $t_1 - t_5$. All specimens are loaded to the same nominal stress σ_a. In all specimens the planes of largest void concentration

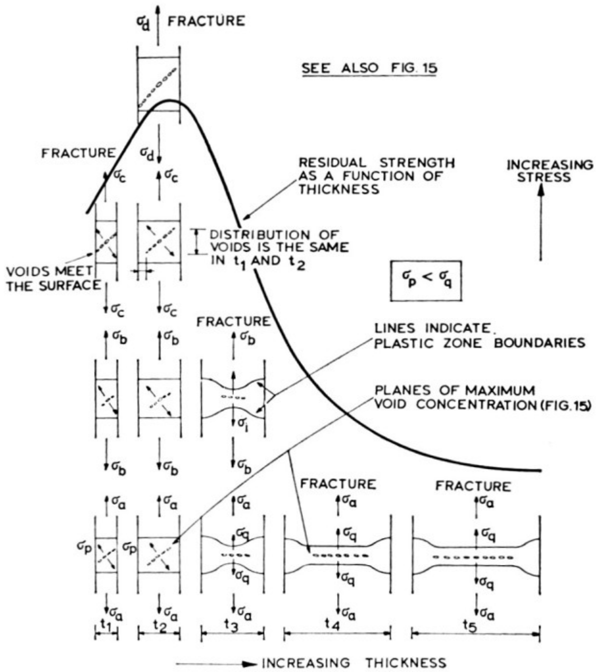


FIG. 17 — The residual strength as a function of thickness (*see also* Fig. 15)

are indicated (*see also* Fig. 15). Also, the size of the plastic zone is indicated which for a certain nominal stress is roughly twice as large in the case of plane stress as in plane strain[13]. As can be seen from Fig. 15(*a*) the tensile stress on the plane of largest void concentration is smaller in plane stress (σ_p) than in-plane strain (σ_q). This indicates that crack growth and eventually fracture will occur in specimens t_4 and t_5 and not yet in specimens t_1 and t_2. The stress in specimens t_1 and t_2 can still be increased. Specimen t_3 is at an intermediate stage.

A speculative explanation for the lower strength of specimen t_1 can be given when it is assumed that the distribution of plastic strain is the same in specimens t_1 and t_2 (Fig. 17). Then also the distribution and size of the voids are the same and then it can be understood that the critical plane in specimen t_1 will be somewhat earlier 'filled in' with voids than in specimen t_2 (Fig. 1).

As a result of the previous considerations one might be tempted to speculate on a criterion for the optimum thickness for a particular material. Different authors[14,15] have proposed that plane stress will be constrained until the size of the plastic zone is at least equal to the plate thickness. Schijve[13] has shown that the size of the plastic zone r_p is given by

$$r_p = C\frac{\sigma^2}{\sigma_y^2}\, l \tag{12}$$

where σ_y is the yield strength. $C \approx 0{\cdot}3$ for plane strain and $C \approx 0{\cdot}6$ for plane stress.

Now consider the data for the 2014 alloy in Fig. 13 and assume that the specimen of 20 mm thickness was the first to have almost completely plane strain. This specimen fractured at a stress $\sigma_c = 22$ kg/mm^2. Then the size of the plastic zone was $r_p = 1{\cdot}35$ mm. The maximum in the curve was at $t = 2$ mm, which was thus the thickness at which full plane stress could develop. This indeed leads to the conclusion that plane stress can develop fully when the plane strain plastic zone size is about equal to the sheet thickness and that this is a reasonable criterion for the optimum thickness.

8. The Residual Strength of Stiffened Panels

Until now there is no satisfactory theoretical method to determine the residual strength of stiffened panels. Only some tentative considerations can be presented here, which are qualitatively supported by test results obtained from large wing panels.

Consider first the simple stiffened panel with two stiffeners in Fig. 18. When this panel contains a crack, which is still small at the onset of instability, the stress condition at the crack tip will hardly be influenced by the stringers and the residual strength will be the same as that of an unstiffened sheet of the same size. Since the crack is small the fracture stress is high and the crack cannot be stopped at the stringers at so high a stress. When the panel contains a crack extending from the left to the right stiffener, the stringers will be the most effective in reducing the peak stress at the crack tips and the residual strength will be much higher than that of an unstiffened panel of the same size with the same length of crack (Fig. 18). When the crack is of an intermediate size, there can be unstable crack growth at a stress slightly above the fracture strength of the unstiffened sheet, due to a small reduction in stress concentration by the stringers. The stress responsible for this unstable crack extension is low enough to make it possible that crack growth is stopped at the stringers, which can be facilitated if the crack runs into rivet holes. The crack then extends from stiffener to stiffener, a case previously considered, and a higher stress is necessary for fracture. Conse-

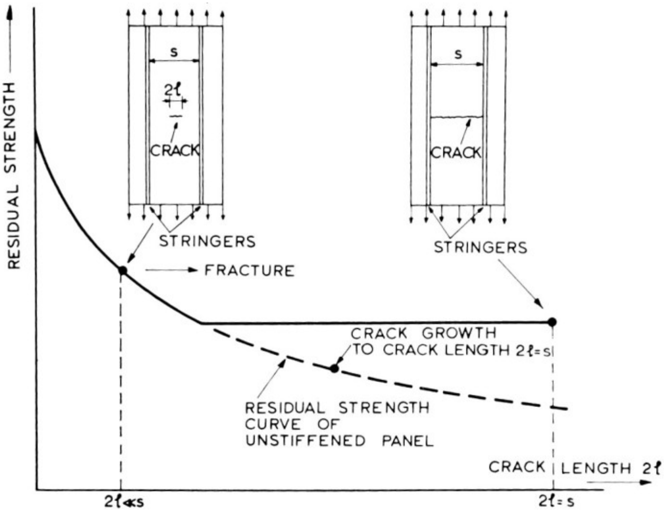


FIG. 18 — Residual strength characteristics of a stiffened panel

quently, a horizontal line can be drawn in the residual strength diagram. Below this line unstable crack growth occurs at such a low stress that crack growth is stopped at the stringers. Above this line the stress is too high and total fracture will occur immediately. Since this case relates to small cracks, the effect of the stringers will be negligible and the residual strength will be the same as that of an unstiffened panel.

These considerations are confirmed qualitatively by test data of large wing panels(16). These panels were 8·5 m (28 ft) long, 1·3 m (4 ft) wide and were stiffened with 11 stringers. The test results are presented in Fig. 19. It is seen that the residual strength of these large panels decreased to a low level by the presence of small cracks. For cracks smaller than about 50 mm complete fracture occurred at stresses in the order of 25 kg/mm^2 and above. At longer cracks unstable crack growth occurred at stresses in accordance with the residual strength curve of an unstiffened sheet, but this crack growth was stopped somewhere at the stringers in a rivet hole or an adhesive layer, and all stringers remained intact. It can be said that probably a stress of the level given by the horizontal line in Fig. 19 would have been required to fracture the panel completely.

At this stage the conclusion can be drawn that the residual strength curve of the unstiffened sheet plays an important role for the residual strength of the stiffened panel. Therefore, the data presented here can fruitfully be used. A test programme to investigate further the residual strength of stiffened panels will be carried out at the National Aerospace Laboratory NLR, Amsterdam. We will also try to develop a calculation method based on residual strength data of unstiffened panels.

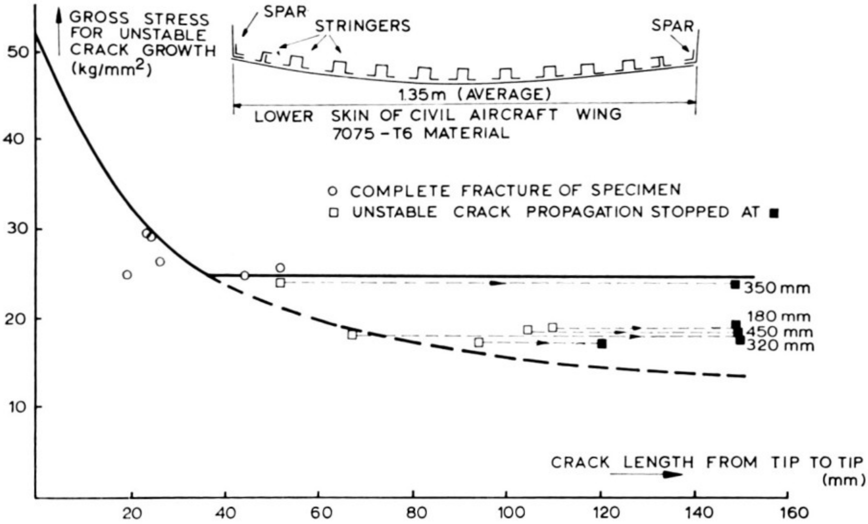


FIG. 19 — Residual strength diagram of wing structure

9. CONCLUSIONS

A survey of the technical knowledge of the residual strength problem has been given, permitting the following conclusions to be drawn:

(*a*) Crack growth can occur when the stress at the tip of the crack exceeds a certain critical value. Crack growth is stable as long as the energy release during crack growth is balanced by the energy consumption.

(*b*) Fracture occurs when the energy release is larger than the energy consumption. This fracture criterion could be evaluated to

$$\sigma_c l_0^{1/2\alpha} = \text{constant},$$

where σ_c is the fracture stress and $2l_0$ the initial crack length. α is a constant which is larger for a more ductile material. The minimum value of α is unity (ideally brittle material).

(*c*) The residual strength can be predicted on the basis of the initial crack length; the critical crack length need not be known. The constant α mentioned above determines the relation between initial crack length and critical crack length. For $\alpha = 1$ there is no slow crack growth (ideally brittle material).

(*d*) Stop holes at the crack tips should have a certain minimum size to have any effect on the residual strength at all. Stop holes increase the stress for the initiation of slow, stable crack growth. When stop holes are so large that the onset of slow crack growth is postponed to a stress exceeding the fracture strength of a specimen with a fatigue crack, immediate fracture occurs at the onset of crack growth. When the stress for the initiation of crack growth is lower than the fracture strength of a specimen with a fatigue crack, the stop holes will have no effect on the residual strength.

(*e*) The residual strength of a cracked sheet depends on the sheet thickness. An optimum residual strength is obtained at a particular thickness.

(*f*) A load release after the occurrence of a small amount of slow crack growth does not impair the residual strength.

(*g*) Residual strength test data of wide unstiffened sheet satisfy the formula given under conclusion (*b*). These data can be of use for the estimation of the residual strength of a stiffened panel.

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APPENDIX

According to ref. 1 the elastic energy of a sheet with a central transverse crack loaded to a stress σ is given by

$$U = \frac{\sigma^2}{2E}(A + 2\pi l^2) \tag{13}$$

per unit thickness. In this formula A represents the area of the sheet (length × width). The equation can be written as

$$U = \frac{\sigma^2}{2E_{\text{eff}}}.A \qquad \text{with} \qquad E_{\text{eff}} = \frac{AE}{A + 2\pi l^2} \tag{14}$$

From eqn. (14) it follows that the effective strain $\bar{\varepsilon}$ is

$$\bar{\varepsilon} = \frac{\sigma}{E_{\text{eff}}} = \frac{\sigma}{E}\left(1 + \frac{2\pi l^2}{A}\right) \tag{15}$$

With the aid of eqn. (15) the elastic energy can be written as a function of $\bar{\varepsilon}$

$$U(\bar{\varepsilon}, l) = \frac{1}{2}\frac{\bar{\varepsilon}^2 EA}{1 + (2\pi l^2/A)} \tag{16}$$

The potential energy of the plate at constant stress can be written as

$$P = U(\bar{\varepsilon}, l) - \sigma\bar{\varepsilon}A \tag{17}$$

per unit thickness.
The potential energy is a minimum as a function of $\bar{\varepsilon}$, hence

$$\frac{\partial P}{\partial\bar{\varepsilon}} = \frac{\partial U(\bar{\varepsilon}, l)}{\partial\bar{\varepsilon}} - \sigma A = 0 \tag{18}$$

From eqns. (17) and (18) (constant stress) it follows that

$$\frac{dP}{dl} = \frac{\partial U(\bar{\varepsilon}, l)}{\partial\bar{\varepsilon}}\frac{d\bar{\varepsilon}}{dl} + \frac{\partial U(\bar{\varepsilon}, l)}{\partial l} - \sigma A\frac{d\bar{\varepsilon}}{dl} = \frac{\partial U(\bar{\varepsilon}, l)}{\partial l} \tag{19}$$

According to eqns. (16) and (19)

$$\frac{dP}{dl} = \frac{\partial U(\bar{\varepsilon}, l)}{\partial l} = -\frac{2\pi\sigma^2 l}{E} \tag{20}$$

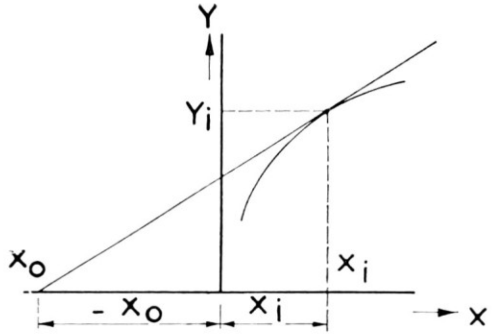

FIG. 20 — The fracture criterion

The fact that

$$l_c = \alpha l_0 \tag{21}$$

for an infinite sheet requires a distinct behaviour of $W'(l)$. Consider Fig. 20, which is the same as Fig. 2, but the axes are denoted by X and Y for convenience. A tangent to the curve in the point x_i, y_i is given by

$$x = y_i + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_i (x - x_i) \tag{22}$$

$x = x_0$ at $y = 0$, hence

$$x_0\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_i = x_i\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_i - y_i \tag{23}$$

and according to eqn. (21)

$$-x_0 + x_i = -\alpha x_0 \tag{24}$$

Combination of eqns. (23) and (24) yields

$$(\alpha - 1)y_i = \alpha x_i\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_i \tag{25}$$

According to eqn. (21) this must be valid for any point i; thus eqn. (8) without the subscript i is the differential equation for the curve in Fig. 20. The solution is

$$y = \beta x^{(\alpha-1)/\alpha} \tag{26}$$

In the notation of Fig. 2 this equation reads

$$\frac{\mathrm{d}W}{\mathrm{d}l} = \beta(l - l_0)^{(\alpha-1)/\alpha} \tag{27}$$